Bonyah *et al. Asia Pac. J. Comput. Engin. (2016) 3:2*
DOI 10.1186/s40540-016-0018-2

Asia Pacific Journal on Computational Engineering

RESEARCH Open Access

Approximate solution of cryptosporidiosis model

Ebenezer Bonyah[1,3*], Abdon Atangana[2], Kazeem Okosun[3] and Muhammad Altaf Khan[4]

*Correspondence:
ebbonya@yahoo.com
[1] Department of Mathematics and Statistics, Kumasi Polytechnic, Kumasi, Ghana
Full list of author information is available at the end of the article

Abstract

Cryptosporidium is associated with waterborne transmission mechanism through the faecal–oral path in many recreational water facilities. We investigate the probable approximate solution of integer and noninteger systems of nonlinear ordinary differential equations representing cryptosporidiosis dynamics. The approximate or estimate solution is derived through recent developed analytic method, the homotopy decomposition method (HDM). The algorithm is systemically explained and demonstrated with some numerical examples. The numerical results indicate that the approximate solution is of continuous function form in the light of noninteger-order derivative. The integer-order numerical solution of parameters values varied and investigated which show similar solution in each case. The method employed to obtain the solution to this problem is robust, easy, reliable and quick in terms of time.

Keywords: Cryptosporidiosis, Homotopy, Gastrointestinal, Decomposition, Noninteger

Background

Human cryptosporidiosis is caused by cryptosporidium protozoan and constitutes a large number of gastrointestinal disease usually connected with recreational water use as the case in Australia [1, 2] as well as other parts of the world [see e.g. [3–6] and references therein]. Cryptosporidiosis is characterized with severe watery diarrhoea; however, asymptomatic infection may arise which becomes the source of infection [1]. Cryptosporidiosis is transmitted through interaction with contaminated water, food and surfaces. Allowing water to get through the mouth into the stomach in recreational swimming is the easiest way of contracting the disease. Crypto is extremely infectious and if not cured one can get the infection again and may infect others. Cryptosporidium is well identified with waterborne transmission mechanism via the faecal–oral path in many recreational water facilities. It is established that the rate of infection is very low in cryptosporidiosis. For instance, it is estimated that it has a low infective dose ranging between 10 and 30 in a healthy adult [7–10]. The disease is capable of resisting to just halogen disinfection which constitutes the recommended level for treating water recreational facilities [11]. If anyone is diagnosed of cryptosporidiosis then the person is likely to have a weak immune system which is a symptom of HIV.

In this paper, we present an SIR model proposed in [25]. The model is characterized by four components which are susceptible human, infected human, recovered human and environment where the bacteria live. Λ is the recruitment rate into the susceptible class. Recovered individual may lose their immunity to the disease at the ω. Recovery rate due to treatment is denoted by σ. The natural and mortality rates due to the disease are μ and ψ, respectively. E_c is the microbe population, and the contact rate of the microbe population is denoted by ν. The concentration of microbe population in the environment is represented by K. The rate of cryptosporidiosis infected to the environment is denoted by π. μ_b is the mortality rate of the microbes. ρ is the rate of contact with the environment

$$\begin{cases} \frac{\mathrm{d}}{dt}S = \Lambda + \omega R - \mu S - \left(\frac{\nu I}{K+I} + \rho E_c\right)S \\ \frac{\mathrm{d}}{\mathrm{d}t}I = \left(\frac{\nu I}{K+I} + \sigma E_c\right)S - (\mu + \psi + \sigma)I \\ \frac{\mathrm{d}}{\mathrm{d}t}R = \sigma I - (\mu + \psi)R \\ \frac{\mathrm{d}}{\mathrm{d}t}E = \pi I - \mu_b E \end{cases} \tag{1}$$

subject to the initial conditions

$$S(0) \geq 0, \quad I(0) \geq 0, R(0) \geq 0 \quad \text{and} \quad E(0) \geq 0.$$

Mathematical models, in general, are highly nonlinear, and obtaining the exact solution usually becomes a challenge. Most researchers resort to numerical solutions. In recent times, there have been several analytical approximation techniques to address these problems. The assistance of computer-aided techniques is growing at a fast rate and numerical simulations have become inevitable. Non-numerical problems are still very crucial because of their role in the socioeconomy of every nation (see e.g. [14–17] references therein). There are several, however, alternative analytical asymptotic techniques which include the nonperturbation modified, Lindstedt–Poincare technique [15], variation iteration technique [17], Adomian decomposition method [18] and homotopy perturbation method [17, 19].

Recently, authors in [13] used HDM to investigate HIV infection of CD4+ T cells and obtained approximate solutions and compared the results with other existing methods. In their study, they found that HDM is as better as other well-known methods as mentioned in the literature. Authors in [20] employed HDM to examine Tuberculosis using both integer and fractional derivative and obtained solutions that are of continuous functions of the noninteger-order derivative. Author in [20] used HDM to investigate cryptosporidiosis model of both integer and fractional order and obtained solutions that are continuous functions of the noninteger-order derivative.

The purpose of this paper is to present approximate analytical solutions for the standard form and fractional aspect of (1) in addition to (2) using the relative new analytical method called homotopy decomposition method (HDM).

The paper is organized as follows: In "Background" section , the basic ideas of homotopy decomposition method are presented. In "Fundamental information about homotopy decomposition method" section, the application of HDM for system for cryptosporidiosis population dynamics is presented. "Stability analysis" section deals with the application of the HDM for the system of fractional cryptosporidiosis model

dynamics. In "Application of the HDM to the model with integer-order derivative" section, the conclusion is drawn.

Fundamental information about homotopy decomposition method

To show the basic notion of this method, we take into consideration a universal nonlinear nonhomogeneous partial differential equation characterized with the following form [12]:

$$\frac{\partial^m U(x,t)}{\partial t^m} = L(U(x,t)) + N(U(x,t)) + f(x,t), \quad m = 1,2,3,\ldots, \tag{2}$$

By focusing on the primary condition, we have

$$\frac{\partial^i (x,0)}{\partial t^i} = y_i, \quad \frac{\partial^m U(x,0)}{\partial t^{m-1}} = 0 \qquad i = 0,1,2,\ldots m-2 \tag{3}$$

where m denotes the order of the derivative, f represents an identified function, N denotes the common nonlinear differential operator, L represents a linear differential operator, and m is the order of the derivative. The initial process here is to ensure that the inverse operator $\partial^m/\partial t^m$ is applied on both sides of (3) so that we obtain [12]

$$U(x,t) = \sum_{k=0}^{m-1} \frac{t^k}{k!} \frac{d^k u(x,0)}{dt^k} + \int_0^t \int_0^{t_1} \cdots \int_0^{t_{m-1}} L(U(x,\tau)) + N(U(x,\tau)) + f(x,\tau)\ldots \mathrm{d}t \tag{4}$$

The multi-integral in Eq. (3) can be reorganized as

$$\begin{aligned} &\int_0^t \int_0^{t_1} \cdots \int_0^{t_{m-1}} L(U(x,\tau)) + N(U(x,\tau)) + f(x,\tau)\ldots \mathrm{d}t \\ &= \frac{1}{(m-1)!} \int_0^t (t-\tau)^{m-1} L(U(x,\tau)) + N(U(x,\tau)) + f(x,\tau)\ldots \mathrm{d}\tau \end{aligned} \tag{5}$$

So, Eq. (3) can be reformulated as

$$\begin{aligned} U(x,t) = &\sum_{k=0}^{m-1} \frac{t^k}{k!} y_i(x) + \frac{1}{(m-1)!} \int_0^t (t-\tau)^{m-1} L(U(x,\tau)) \\ &+ N(U(x,\tau)) + f(x,\tau)\ldots \mathrm{d}t \end{aligned} \tag{6}$$

Employing the homotopy scheme, the solution of the above-mentioned integral equation is expressed in series form as follows:

$$\begin{aligned} U(x,t) &= \sum_{k=0}^{m-1} p^m U_n(x,t), \\ U(x,t) &= \lim_{p\to 1} U(x,t,p) \end{aligned} \tag{7}$$

and the nonlinear term can be decomposed by

$$\mathrm{NU}(r,t) = \sum_{n=1}^{\infty} P^n \Re_n(U),$$

where $p \in (0,1]$ denotes an implanting parameter and $\Re_n(U)$ represents the polynomial that can be engendered by

$$\Re_n(U)(U_0 \ldots, U_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{j=0}^{n} p^j U_j(x,t)\right)\right] \quad n = 0,1,2,3 \tag{8}$$

The homotopy decomposition method is composed of the decomposition method and Abel integral which is given as

$$\sum_{n=0}^{\infty} p^n U_n(x,t) = \mathrm{T}(x,t) + p\frac{1}{(m-1)!} \times \int_0^1 (t-\tau)^{m-1}\left[f(x,t) + \left(\sum_{n=0}^{\infty} p^n U_n(x,t)\right) + \sum_{n=0}^{\infty} p^n \Re_n(U)\right] \mathrm{d}\tau \tag{9}$$

with

$$\mathrm{T}(x,t) = \sum_{k=0}^{m-1} \frac{t^k}{k!} y_i(x).$$

Matching the term of identical powers of p_i leads to solution of different orders. The estimate of the approximation is $\mathrm{T}(x,t)$ that is precisely the Taylor series presenting the exact solution of order m. It is worthy to note that the initial guess or estimate assures the uniqueness of the series decompositions [20, 21].

Stability analysis

The stability analysis of a model is very essential, which allows to establish the behaviour of the model. The disease-free equilibrium is obtained by setting system 1 to be equal to zero and solving

$$E_0 = \left(S^*, I^*, R^*, E^*\right) = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right).$$

The linear stability of E_0 is determined by applying the next-generation operator technique [26] on system 1, and the reproduction number is obtained as follows:

$$R_0 = \frac{\nu\Lambda\sqrt{\mu_b} + \sqrt{\nu^2\Lambda^2\mu_b + 4K^2\pi\mu^2\rho(\mu+\sigma+\psi))}}{2K\mu\sqrt{\mu_b}(\mu+\sigma+\psi)}.$$

Theorem 1 *The disease free equilibrium of the model 1, given by R_0 , is locally asymptotically stable if $R_0 < 1$, and unstable if $R_0 > 1$.*

Proof

$$\begin{cases} S^* = \frac{\Lambda+\omega R^*}{\mu+\beta^*}, \\ R^* = \frac{\sigma I^*}{\omega+\mu}, \\ E^* = \frac{\pi I^*}{\mu_b}. \end{cases} \tag{10}$$

$$q(I^*) = I^*\Big(G_1(I^*)^2 + G_2(I^*) + G_3\Big) = 0 \tag{11}$$

$$G_1 = \rho\rho(\mu(\mu+\sigma+\psi) + (\mu+\psi)\psi),$$

$$G_2 = \frac{K\mu_b\mu(\mu+\sigma+\psi)}{\nu\mu_b + K\rho\rho}\big(R_p - R_0\big),$$

$$G_3 = K\mu_b\mu(\mu+\omega)(\mu+\sigma+\psi)(1-R_0)$$

$$R_p = \frac{(\nu\mu_b + K\rho\rho)(K\rho\rho[\mu^2 + (\mu+\psi)(\sigma+\psi)] + \mu\mu_b(\nu+\mu)(\mu+\sigma+\psi+Q))}{K\mu\mu_b(\mu+\sigma+\psi)}$$

Theorem 2 *For $K = 0$, the system 1 has no endemic equilibrium and for $K > 0$ the model exhibits two conditions: a transcritical bifurcation if $R_p \geq 1$ and a backward bifurcation if $R_p < 1$.*

□

For more details about the stability analysis of system 1, see [20] reference therein.

Application of the HDM to the model with integer-order derivative

This section discusses the usage of HDM to derive the set of mathematical equations based on cryptosporidiosis population dynamics model. Following the steps involved in HDM method, we arrive at the following integral equation;

$$\begin{aligned} p^0 &= S_0(t) = S(0), \\ p^0 &= I_0(t) = I(0), \\ p^0 &= R_0(t) = R(0), \\ p^0 &= E_0(t) = E(0) \end{aligned} \tag{12}$$

$$\begin{aligned} p^1 : S_1(t) &= \int_0^t (\Lambda + \omega R_0(\tau) - \mu S_0(\tau) - \Big(\tfrac{\nu I_0}{K+I_0} + \rho E_{c0}\Big)S_0(\tau))d\tau,\ S_0(0) = 0, \\ p^1 : I_1(t) &= \int_0^t \Big(\tfrac{\nu I_0}{K+I_0} + \sigma E_{c0}\Big)S_0(\tau) - (\mu+\psi+\sigma)I_0(\tau))\mathrm{d}\tau, \quad \mathrm{I_0(0) = 0}, \\ p^1 : R_1(t) &= \int_0^t \sigma I_0(\tau) - (\mu+\psi)R_0(\tau))\mathrm{d}\tau, \quad \mathrm{R_0(0) = 0}, \\ p^1 : E_1(t) &= \int_0^t \pi I_0(\tau) - \mu_b E_0(\tau))\mathrm{d}\tau, \quad \mathrm{E_0(0) = 0}. \end{aligned} \tag{13}$$

$$
\begin{aligned}
p^n : S_n(t) &= \int_0^t (\Lambda + \omega R_{(n-1)}(\tau) - \mu S_{(n-1)}(\tau) \\
&\quad - \sum_{j=0}^{n-1} \left(\frac{\nu I_j}{K + I_j} + \rho E_{cj} \right) S_{n-j-1}(\tau) d\tau, \quad S_{n-1}(0) = 0, \\
p^n : I_n(t) &= \int_0^t \left(\sum_{j=0}^{n-1} \left(\frac{\nu I_j}{K + I_j} + \sigma E_{cj} \right) S_{n-j-1}(\tau) - (\mu + \psi + \sigma) I_{n-1}(\tau) \right) d\tau, \quad I_{n-1}(0) = 0, \\
p^n : R_n(t) &= \int_0^t (\sigma I_{n-1}(\tau) - (\mu + \psi) R_{n-1}(\tau) d\tau, \quad R_{n-1}(0) = 0, \\
p^n : E_n(t) &= \int_0^t (\pi I_{n-1}(\tau) - \mu_b E_{n-1}(\tau))) d\tau, \quad E_{n-1}(0) = 0.
\end{aligned} \tag{14}
$$

Using integration techniques, we arrived at the following components:

$$
\begin{aligned}
&S_0(t) = S(0); I_0(t) = I(0); R_0(t) = R(0); E_0(t) = E(0) \\
&S_1(t) = \left(\Lambda + \omega R_0 - \mu S_0 - \left(\tfrac{\nu I_0}{K + I_0} + \rho E_{c0} \right) S_0 \right) t, \\
&I_1(t) = \left(\left(\tfrac{\nu I_0}{K + I_0} + \sigma E_{c0} \right) S_0 - (\mu + \psi + \sigma) I_0 \right) t, \\
&R_1(t) = (\sigma I_0 - (\mu + \psi) R_0) t, \\
&E_1(t) = (\pi I_0 - \mu_b E_0) t.
\end{aligned} \tag{15}
$$

For the purpose of simplification, we let

$$
\begin{aligned}
a_0 &= \left(\Lambda + \omega R_0 - \mu S_0 - \left(\tfrac{\nu I_0}{K + I_0} + \rho E_{c0} \right) S_0 \right), \\
b_0 &= \left(\left(\tfrac{\nu I_0}{K + I_0} + \sigma E_{c0} \right) S_0 - (\mu + \psi + \sigma) I_0 \right), \\
c_0 &= (\sigma I_0 - (\mu + \psi) R_0), \\
d_0 &= (\pi I_0 - \mu_b E_0).
\end{aligned} \tag{16}
$$

By applying integrating $S_1(t), I_1(t), R_1(t)$ and $E_1(t)$, we have

$$
\begin{aligned}
S_2(t) &= \frac{t^2}{2} \left(+\omega c - \mu a - \frac{\nu I_0}{K + I_0} a - \frac{\nu b}{K + b} S_0 - a \rho E_{c0} - \rho d S_0 \right) \\
&= S_2(t) = \frac{t^2}{2} a_1 \\
I_2(t) &= \frac{t^2}{2} \left(\frac{\nu I_0}{K + I_0} a + \frac{\nu b}{K + b} S_0 + \rho E_{c0} + \rho d S_0 - (\mu + \psi + \sigma) b \right) \\
&= I_2(t) = \frac{t^2}{2} b_1 \\
R_2(t) &= \frac{t^2}{2} (\sigma a - (\mu + \psi) c) \\
&= R_2(t) = \frac{t^2}{2} c_1 \\
E_2(t) &= \frac{t^2}{2} (\pi a - \mu_b d) \\
&= E_2(t) = \frac{t^2}{2} d_1
\end{aligned}
$$

In general, the following recursive formulas are obtained:

$$\begin{aligned} S_n(t) &= \tfrac{t^n}{n!}a_n, \\ I_n(t) &= \tfrac{t^n}{n!}b_n, \\ R_n(t) &= \tfrac{t^n}{n!}c_n, \\ E_n(t) &= \tfrac{t^n}{n!}d_n, \end{aligned} \tag{17}$$

where a_n, b_n, c_n and d_n rely on the fixed set of empirical parameters. It thus, in principle, implies that the approximate solution of the system (1) is obtained as

$$\begin{aligned} S_n(t) &= \sum_{n=0}^{N} \tfrac{t^n}{n!}a_n, \\ I_n(t) &= \sum_{n=0}^{N} \tfrac{t^n}{n!}b_n, \\ R_n(t) &= \sum_{n=0}^{N} \tfrac{t^n}{n!}c_n, \\ E_n(t) &= \sum_{n=0}^{N} \tfrac{t^n}{n!}d_n \end{aligned} \tag{18}$$

The total human population is assumed to be 140; the initial susceptible human population is 120; the initial infected human population is 5 and initial human recovered population 15. The initial cryptosporidiosis population is 12. The human recruitment rate is $\Lambda = 0.0004$; individual may lose their immunity to the disease at $\omega = 0.001$; recovery rate due to treatment is $\sigma = 0.07$; natural mortality is $\mu = 0.00055$; mortality rate due to the disease is $\psi = 0.006$; contact rate of the microbe population is $\nu = 0.5$; the microbe population is $K = 1000$; cryptosporidiosis infection contribution to the environment is $\pi = 0.045$; mortality rate of the microbe is $\mu_b = 0.033$ and the rate of humans contact with the environment is $\rho = 0.045$.

We make an assumption that there is a constant population. Figure 1 depicts the approximate solution of the system (1). In Fig. 1, the susceptible population with time decreases as more humans get infected and others die due to natural death. This is envisaged in biologically feasible situation. The infected humans in Fig. 1 increase in a short period and then decrease. This could be the fact that people get to know about the disease and try to avoid making contact with the microbe. The recovered humans appeared to reduce and could be attributable to difficulty in diagnosing a patient with cryptosporidiosis even in the advanced world. The microbe population decreases with time and this could be the fact that health authorities may design a problem to fight against the microbe. We varied some of the parameters in Fig. 2 in order to observe the dynamics in the integer-order situation. In Fig. 2, susceptible, recovered and microbe population are quickly moving towards the origin. However, the infective population in Fig. 2 is relatively slow moving towards the origin. This is expected as susceptible humans get infected and then slowly the infected get recovered. It can be seen from Fig. 2 that the microbe population also reduces because people get the awareness and prevent themselves.

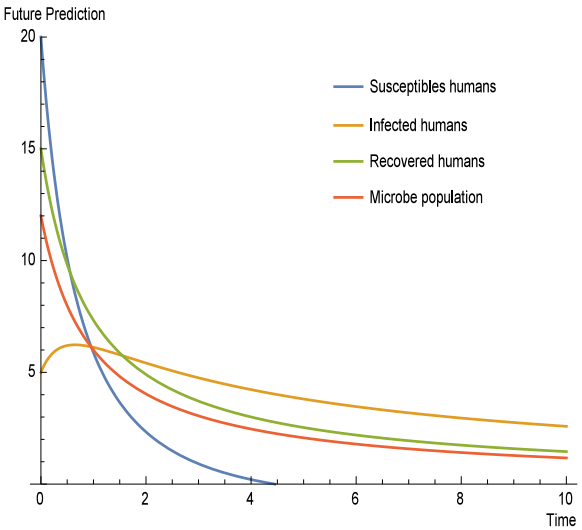

Fig. 1 Approximate solution for $\alpha = 1, \beta = 1, \gamma = 1, \xi = 1$

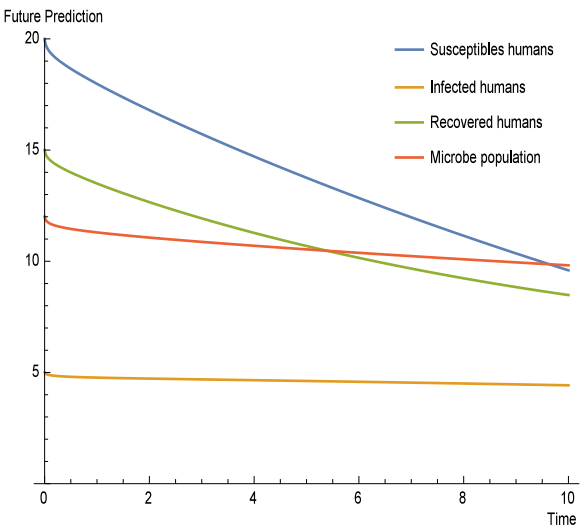

Fig. 2 Approximate solution for $\alpha = 1, \beta = 1, \gamma = 1, \xi = 1$

Application of the HDM to the model with noninteger-order derivative

In recent times, fractional calculus has been identified as a powerful to tool to model many physical and engineering processes, which are best described in terms of fractional differential equation [12]. It is remarkable to note that the usual standard mathematical models in the form of integer-order derivatives fall short of vividly describing many physical problems. Fractional calculus for the past few years has become indispensable in many field of endeavour which include mathematics, biology, chemistry, food science, mechanics, electricity, electronics, image processing, control theory and many more. Some of the vital topics include fractional filters, computational fractional derivative equations, nonlocal phenomena; porous media, biomathematics and fractional phase-locked loops (see [19, 22–24]).

Properties and definitions

Definition 1 A real function $f(x), x > 0$, is said to be in the space $C_\mu, \mu \in \mathbb{R}$, if there exists a real number $p > \mu$, such that $f(x) = x^p h(x)$, where $h(x) \in C[0, \infty)$, and it is said to be in space C_μ^m if $f^{(m)} \in C_\mu, m \in \mathbb{N}$

Definition 2 The Riemann–Liouville fractional integral operator of order $\alpha \geqslant 0$, of a function $f \in C_\mu, \mu \geqslant -1, \alpha, \beta \geqslant 0$, is defined as follows:

$$J^\alpha J^\beta f(x) = J^{\alpha+\beta} f(x)$$

$$J^\alpha J^\beta f(x) = J^\beta J^\alpha f(x) J^\alpha x^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} x^{\alpha+\gamma}.$$

Lemma 1 *If* $m-1 < \alpha \leqslant m, m \in \mathbb{N}$ *and* $f \in C_\mu^m, \mu \geqslant 1$, *then*

$$D^\alpha J^\alpha f(x) = f(x),$$

$$J^\alpha D^\alpha f(x) = f(x) - \sum_{k=0}^{m-1} f^{(k)}(0^+) \frac{x^k}{k!}; x > 0$$

Definition 3 (*partial derivatives of fractional order*). Assume now that $f(x)$ is a function of n variables $x_i i = 1, \ldots, n$ also of class C on $D \in \mathbb{R}_n$. We define partial derivative of order α for f respect to x_i the function

$$a\partial_{\underline{x}}^\alpha f = \frac{1}{\Gamma(m-\alpha)} \int_a^{x_j} (x_i - t)^{m-\alpha-1} a\partial_{x_i}^m f(x_j)\Big|_{x_j=t} \mathrm{d}t$$

where $\partial_{x_i}^m$ is the usual partial derivative of integer-order m.

Approximate solution of fractional version

The system 1 that is transformed into fractional derivative is expressed as

$$\begin{cases} \frac{\mathrm{d}^\theta}{\mathrm{d}t^\theta} S = \Lambda + \omega R(t) - \mu S(t) - \left(\frac{\nu I}{K+I} + \rho E_c\right) S(t), \quad 0 < \theta \leq 1 \\ \frac{\mathrm{d}^\vartheta}{\mathrm{d}t^\vartheta} I = \left(\frac{\nu I}{K+I} + \sigma E_c\right) S(t) - (\mu + \psi + \sigma) I(t), \quad 0 < \vartheta \leq 1 \\ \frac{\mathrm{d}^\upsilon}{\mathrm{d}t^\upsilon} R = \sigma I(t) - (\mu + \psi) R(t), \quad 0 < \upsilon \leq 1 \\ \frac{\mathrm{d}^\varpi}{\mathrm{d}t^\varpi} E = \pi I - \mu_b E, \quad 0 < \varpi \leq 1 \end{cases} \tag{19}$$

Following the discussion presented earlier, we arrive at the following equations:

$$\begin{aligned} p^0 &= S_0(t) = S(0), \\ p^0 &= I_0(t) = I(0), \\ p^0 &= R_0(t) = R(0), \\ p^0 &= E_0(t) = E(0) \end{aligned} \tag{20}$$

$$p^1 : S_1(t) = \frac{1}{\Gamma(\theta)} \int_0^t (t-\tau)^{\theta-1} \times (\Lambda + \omega R_0(\tau) - \mu S_0(\tau) - \left(\frac{\nu I_0}{K + I_0} + \rho E_{c0}\right) S_0(\tau), \Big) d\tau \quad S_1(0) = 0,$$

$$p^1 : I_1(t) = \frac{1}{\Gamma(\vartheta)} \int_0^t (t-\tau)^{\vartheta-1} \times \left(\left(\frac{\nu I_0}{K + I_0} + \sigma E_{c0} \right) S_0(\tau) - (\mu + \psi + \sigma) I_0(\tau) \right) d\tau, \quad I_1(0) = 0,$$

$$p^1 : R_1(t) = \frac{1}{\Gamma(\upsilon)} \int_0^t (t-\tau)^{\upsilon-1} \times (\sigma I_0(\tau) - (\mu + \psi) R_0(\tau)) d\tau, \quad R_1(0) = 0,$$

$$p^1 : E_1(t) = \frac{1}{\Gamma(\varpi)} \int_0^t (t-\tau)^{\varpi} \times (\pi I_0(\tau) - \mu_b E_0(\tau)) d\tau, \quad E_1(0) = 0.$$

$$p^n : S_n(t) = \frac{1}{\Gamma(\theta)} \int_0^t (t-\tau)^{\theta-1} \times \left(\Lambda + \omega R_{(n-1)}(\tau) - \mu S_{(n-1)}(\tau) - \sum_{j=0}^{n-1} \left(\frac{\nu I_j}{K + I_j} + \rho E_{cj} \right) S_{n-j-1}(\tau) \right) d\tau, \quad S_{n-1}(0) = 0,$$

$$p^n : I_n(t) = \frac{1}{\Gamma(\vartheta)} \int_0^t (t-\tau)^{\vartheta-1} \times \left(\sum_{j=0}^{n-1} \left(\frac{\nu I_j}{K + I_j} + \sigma E_{cj} \right) S_{n-j-1}(\tau) \right) - (\mu + \psi + \sigma) I_{n-1}(\tau)) d\tau, I_{n-1}(0) = 0$$

$$p^n : E_n(t) = \frac{1}{\Gamma(\upsilon)} \int_0^t (t-\tau)^{\upsilon-1} \times (\pi I_{n-1}(\tau) - \mu_b E_{n-1}(\tau))) d\tau, \quad E_{n-1}(0) = 0,$$

$$p^n : R_n(t) = \frac{1}{\Gamma(\varpi)} \int_0^t (t-\tau)^{\varpi-1} \times (\sigma I_{n-1}(\tau) - (\mu + \psi) R_{n-1}(\tau)) d\tau, \quad R_{n-1}(0) = 0.$$

The figures depict the solutions obtained for different values of the fractional order derivatives. The approximate solutions of the system (1) are shown in Figs. 2, 3, 4 and 5, 6, 7 respectively. It is obvious from Figs. 2, 3, 4 and 5 that the fractional derivatives

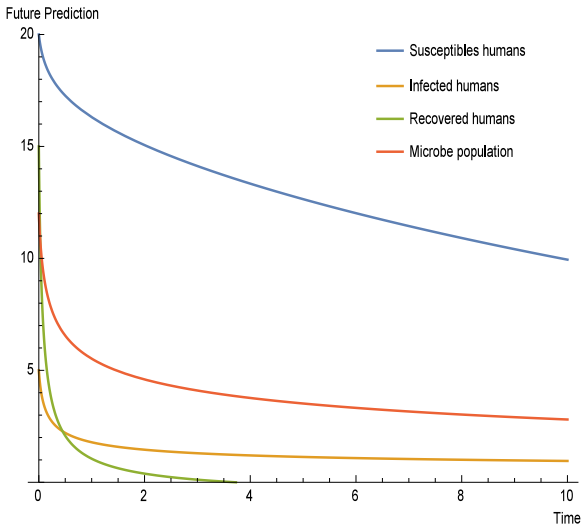

Fig. 3 Approximate solution for $\alpha = 0.5, \beta = 0.65, \gamma = 0.25, \xi = 0.75$

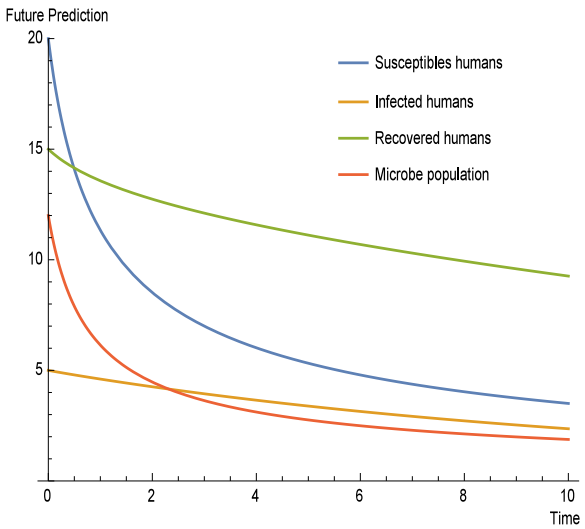

Fig. 4 Approximate solution for $\alpha = 0.5, \beta = 0.95, \gamma = 0.95, \xi = 0.35$

approximate solutions far better than that of Fig. 1, which is the integer-order derivative approximate solution of the system (1). In Fig. 1, the solutions are quickly converging towards the time axes but in Figs. 2, 3, 4 and 5 one can observe a continuous solution as they move away from being asymptotic to the time axes. The numerical solutions indicate that the approximate solutions are of continuous functions in character of the noninteger-order derivatives. It is worthy of interest to note that the usual mathematical

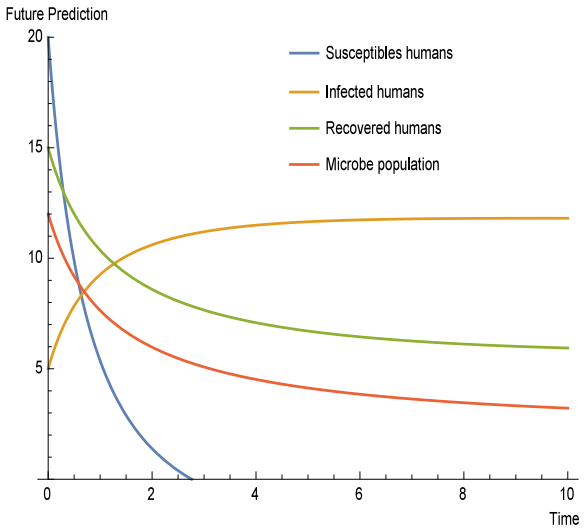

Fig. 5 Approximate solution for $\alpha = 0.02, \beta = 0.5, \gamma = 0.5, \xi = 0.95$

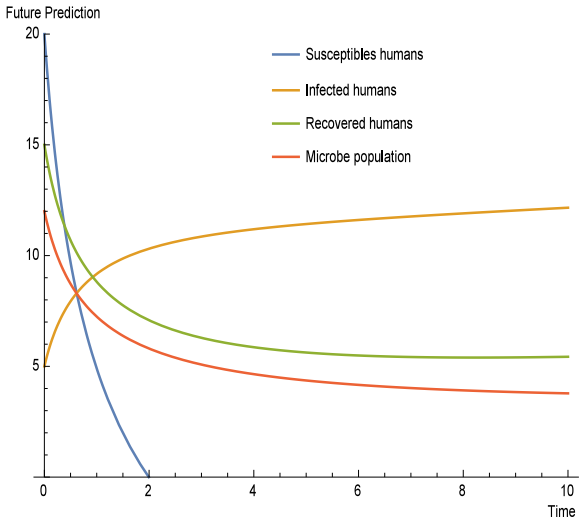

Fig. 6 Approximate solution for $\alpha = 0.5, \beta = 0.5, \gamma = 0.5, \xi = 0.5$

models of integer-order derivatives such nonlinear models usually fall short in many instance in terms of vivid description of the situation. It is prudent, therefore, to entreat people to make use of fractional models in order to vividly represent problems as it is in their natural settings.

Conclusion

The cryptosporidiosis model presented in this paper was investigated in the instances of both integer- and noninteger-order derivatives perspective. The model was solved using a recently developed and iterative technique called homotopy decomposition

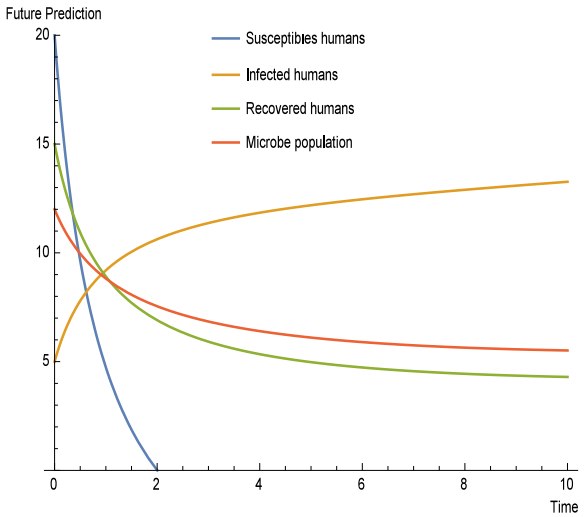

Fig. 7 Approximate solution for $\alpha = 0.2, \beta = 0.04, \gamma = 0.35, \xi = 0.6$

method. The detailed fundamental characteristics of HDM are presented. The noninteger approximate solutions turn to be increasing continuous functions of the fractional derivative. The algorithm for cryptosporidiosis models is very friendly, effective, simple, reliable and quick. The numerical results for the both instances exhibit the real biological dynamics of the problem solved.

Authors' contributions
The authors have contributed equally for the production of this manuscript. All authors read and approved the final manuscript.

Author details
[1] Department of Mathematics and Statistics, Kumasi Polytechnic, Kumasi, Ghana. [2] Institute for Groundwater Studies, University of the Free State, Bloemfontein 9301, South Africa. [3] Department of Mathematics, Vaal University of Technology, Vanderbijlpark, South Africa. [4] Department of Mathematics, Abdul Wali Khan University Mardan, Khyber Pakhtunkhwa, Pakistan.

Competing interests
The authors declare that they have no competing interests.

Received: 20 July 2016 Accepted: 28 October 2016
Published online: 11 November 2016

References

1. Heymann DL (2008) Control of communicable diseases manual: an official report of the American Public Health Association, 19th edn. American Public Health Association, Washington
2. Black M, McAnulty J (2006) The investigation of an outbreak of cryptosporidiosis in New South Wales in 2005. N S W Public Health Bull 17(5–6):7679
3. Boehmer TK, Alden NB, Ghosh TS, Vogt RL (2009) Cryptosporidiosis from a community swimming pool: outbreak investigation and follow-up study. Epidemiol Infect 137(11):16511654
4. Centers for Disease Control and Prevention (2007) Cryptosporidiosis outbreaks associated with recreational water usefive states 2006. Morbidity and Mortality Weekly Report 56(29):729732
5. Insulander M., Lebbad M., Stenstrom T. A., Svenungsson B. *An outbreak of cryptosporidiosis associated with exposure to swimming pool water,* (Scandinavian Journal of Infectious Diseases, vol. 37, no. 5, pp. 354360, 2005.)
6. McAnulty JM, Fleming DW, Gonzalez AH (1994) A community- wide outbreak of cryptosporidiosis associated with swimming at a wave pool. Journal of the American Medical Association 272(20):15971600
7. Puech MC, McAnulty JM, Lesjak M, Shaw N, Heron L, Watson JM (2001) A state wide outbreak of cryptosporidiosis in New South Wales associated with swimming at public pools. Epidemiol Infect 126(3):389396
8. Lemmon J.M., McAnulty J.M., Bawden-Smith J. *Outbreak of cryptosporidiosis linked to an indoor swimming pool,* (Medical Journal of Australia, vol. 165, no. 11-12, pp. 613616, 1996.)

9. Chappell CL, Okhuysen PC, Sterling CR, DuPont HL (1996) Cryptosporidium parvum: intensity of infection and oocyst excretion patterns in healthy volunteers. Journal of Infectious Diseases 173(1):232236
10. Jokipii L, Jokipii AM (1986) Timing of symptoms and oocyst excretion in human cryptosporidiosis. N Engl J Med 315(26):16431647
11. Korich DG, Mead JR, Madore MS, Sinclair NA, Sterling CR (1990) Effects of ozone, chlorine dioxide, chlorine, and monochloramine on Cryptosporidium parvum oocyst viability. Appl Environ Microbiol 56(5):14231428
12. Atangana A, Necdet B (2013) Approximate solution of tuberculosis disease population dynamics model. Abstr Appl Anal 2013(2013):759801
13. Atangana A, Goufo EFD (2014) Computational analysis of the model describing HIV infection of CD4+T cells. Biomed Res Int 2014(2014):7
14. Andrianov I., Awrejcewicz J. *Construction of periodic +solutions to partial differential equations with non-linear boundary conditions,* (International Journal of Nonlinear Sciences and Numerical Simulation, vol. 1, no. 4, pp. 327332, 2000.)
15. Bender CM, Milton KA, Pinsky SS, Simmons J (1989) A new perturbative approach to nonlinear problems. J Math Phys 30(7):14471455
16. Delamotte B (1993) Nonperturbative method for solving differential equations and finding limit cycles. Phys Rev Lett 70(22):33613364
17. Wu G, Baleanu D (2013) Variational iteration method for fractional calculus – a universal approach by Laplace transform. Adv Difference Equ 2013(1):1–9
18. Adomian G (1988) A review of the decomposition method in applied mathematics. J Math Anal Appl 135(2):501544
19. Atangana A, Secer A (2013) The time-fractional coupled- Korteweg-de-Vries equations. Abstr Appl Anal 2013(8):947986
20. Suares C, Oukouomi N (2013) Homotopy decomposition analysis of a tuberculosis model. Life Sci J 10(3):459–465
21. Atangana A, Botha JF (2012) Analytical solution of the groundwater flow equation obtained via homotopy decomposition method. (J. Earth Sci. Climate Change 3:115. doi:10.4172/2157-7617.1000115
22. Oldham KB, Spanier J (1974) The fractional calculus. Academic Press, New York
23. Podlubny I (1999) Fractional differential equations. Academic Press, New York
24. Caputo M (1967) Linear models of dissipation whose Q is almost frequency independent, part II. Geophys J Int 13(5):529–539
25. Okosun KO, Mukamuri M, Makinde OD (2016) Co-dynamics of trypanosomiasis and cryptosporidiosis. Appl Math Inform Sci 10(6):2137–2161
26. Van den Driessche P, Watmough J (2002) Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Math Biosci 180(1):29–48