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Double stratification effects on heat and mass transfer in unsteady MHD nanofluid flow over a flat surface

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Abstract

The focus of this work is to theoretically investigate the problem of double stratification on heat and mass transfer in an unsteady hydromagnetic boundary layer flow of a nanofluid over a flat surface. The model employed for the nanofluid transport equations incorporate the effects of Brownian motion and thermophoresis in the presence of thermal and solutal stratification. The governing nonlinear partial differential equations and their associated boundary conditions are initially transformed into dimensionless form by using similarity variables, before being solved numerically by employing the Runge–Kutta–Fehlberg fourth-order method with shooting technique. The effects of different controlling parameters, viz. solutal and thermal stratification, Lewis number, thermophoresis, Brownian motion, magnetic field and unsteadiness on the fluid velocity, temperature, skin friction coefficient, the local Nusselt number, and the local Sherwood number are graphically depicted and quantitatively discussed in detail taking into account the practical applications of each profile. It is noted that thermal stratification reduces the fluid temperature, while the solutal stratification reduces the nanoparticle concentration.

Keywords: Heat transfer, Mass transfer, Unsteady, MHD, Nanofluid, Double stratification

Background

Magnetohydrodynamics (MHD) boundary layer flow of electrically conducting fluids has diverse industrial and engineering applications in fields such as nuclear reactors, geothermal engineering, liquid metals and plasma flows, petroleum industries, boundary layer control in aerodynamics and crystal growth. Several authors have studied the problem of MHD boundary layer flow, heat and mass transfer about different surface geometries in electrically conducting fluids. Makinde [1] numerically analysed the influence of magnetic field on the steady heat and mass flow of an electrically conducting fluid by mixed convection along a semi-infinite vertical porous plate with constant heat flux taking into account Soret and Dufour effects. The effect of the Hall current on the MHD natural convection flow from a vertical permeable flat plate with a uniform heat flux in the presence of a transverse magnetic field was analysed by Saha et al. [2]. Rout et al. [3] investigated MHD heat and mass transfer of chemically reacting fluid flow over a moving vertical plate in the presence of a heat source with convective boundary condition.

Stratification is the layering of a fluid system due to temperature differences and variations in concentration or the presence of different fluids with varying densities. In practical situations where the heat and mass transfer mechanisms occur simultaneously, it is important to analyse the effect of double stratification (stratification of medium with respect to thermal and concentration fields) on the convective transport in the fluid. The effect of double stratification is important in applications involving fluids' convective transport where heat and mass transfer run concurrently. It is encountered in several classical problems, for example, reservoir mixing, internal waves, shear flow instability, internal hydraulics, turbulence and jets, plumes and wakes. Stratified fluids are omnipresent in nature and their occurrence is very common in heterogeneous fluid bodies, such as thermal stratification of reservoirs and oceans, salinity stratification in estuaries, rivers, groundwater reservoirs, and oceans, heterogeneous mixtures in industrial food manufacturing processing, density stratification of the atmosphere, and uncountable similar examples [4]. For instance, density variation causes thermal stratification in reservoirs which leads to a reduction in the vertical mixing of oxygen to the point that bottom water becomes anoxic through the action of biological processes. Preventing, predicting, and solving such a reservoir problem, though dependent on other limnological factors, require an understanding of the dynamics of stratified fluids. MHD-convective heat transfer in thermally stratified fluid occurs in many industrial applications and is an important aspect in the study of heat transfer. Higher energy efficiency and increased system performance can be achieved with better thermal stratification. This has led to intensified research in mass and heat transfer in thermally stratified medium [5–7]. However, the aforementioned investigations analysed natural convection flow of convective heat transfer fluids.

Recently with the advent of nanofluids—a term coined by Choi [8] referring to a liquid containing a suspension of nanometre-sized solid particle (nanoparticles)—research has diversified to convective heat and mass transfer of nanofluids in doubly stratified medium. Although the effect of double stratification on heat and mass transfer in a fluid is significant, very little work has been reported in the literature [9–11]. On the other hand, studies on unsteady boundary layer flow, mass transfer, and convective heat transfer in a nanofluid are the topics of contemporary research areas in fluid science and engineering, owing to its novelty [12–17]. The mass and heat transfer in a doubly stratified medium in the presence of a magnetic field is not only fundamentally interesting but also finds wide range of natural, industrial, and engineering applications. These applications include heat rejection into the environment such as lakes, rivers, and seas; thermal energy storage systems such as solar ponds; heat transfer from thermal sources such as condensers of power plants.

Despite its numerous applications, it is evident from the literature herein that research has neglected mass and heat transfer in unsteady MHD nanofluid flow taking into account thermophoresis and Brownian motion in a doubly stratified medium. Hence, the motivation of this paper is to bridge this knowledge gap. This study aims to extend the recent work of Olanrewaju and Makinde [18] to incorporate unsteadiness, magnetic field, and the effects of double stratification on boundary layer nanofluid flow. The shooting iteration technique coupled with fourth-order Runge–Kutta integration scheme is employed to numerically solve the model nonlinear system of equations of

this particular problem. Numerical results are displayed graphically and discussed for different values of the various dimensionless parameters controlling the flow regime. It is hoped that the results obtained will not only provide useful information for applications but also serve as a basis for studying other analogous systems that arise in engineering and industrial applications.

Model formulation

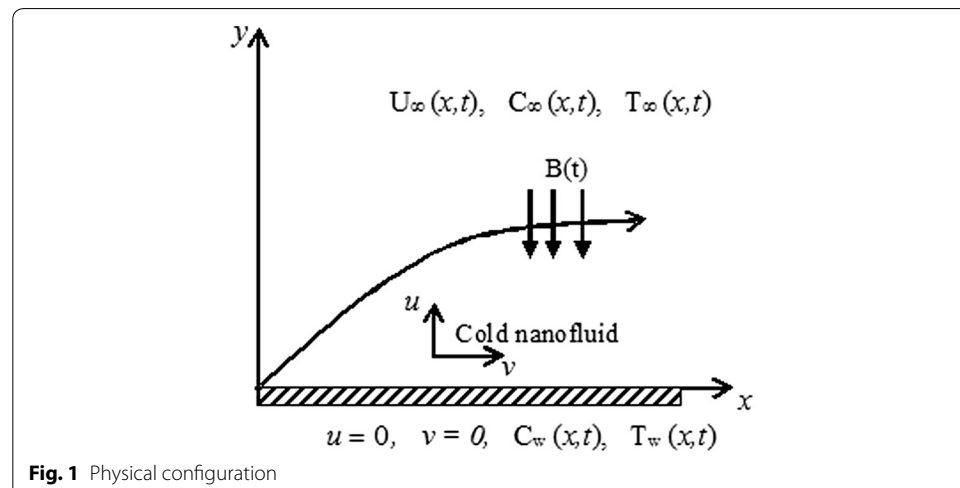
We consider heat and mass transfer in an unsteady flow of an incompressible electrically conducting viscous nanofluid past an infinite nonconducting flat plate in the presence of a transversely imposed unsteady magnetic field of strength $B(t)$. The sketch of the physical configuration and coordinate system is shown in Fig. 1. The induced magnetic field is neglected, which is justified for industrial application MHD flows at small magnetic Reynolds number. It is also assumed that the external electrical field is zero and that the electric field due to the polarization of charges is also negligible. The temperature and nanoparticle concentration at the plate surface and at the free stream are assumed to vary with time and axial distance along the plate.

Under the above assumptions with the boundary layer approximations, the continuity, momentum, energy, and nanoparticle concentration equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial U_\infty}{\partial t} + U_\infty \frac{\partial U_\infty}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B(t)^2 (u - U_\infty)}{\rho}, \quad (2)$$

$$\begin{aligned} \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = & \frac{\partial T_\infty}{\partial t} + U_\infty \frac{\partial T_\infty}{\partial x} + \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 \\ & - \frac{\sigma B(t)^2 (u - U_\infty)^2}{\rho c_p} + \tau \left\{ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_{0,\infty}} \left(\frac{\partial T}{\partial y} \right)^2 \right\}, \end{aligned} \quad (3)$$



$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{\partial C_\infty}{\partial t} + U_\infty \frac{\partial C_\infty}{\partial x} + D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_{0,\infty}} \frac{\partial^2 T}{\partial y^2}, \quad (4)$$

with the boundary conditions

$$\begin{aligned} u = 0, v = 0, T = T_w(x, t), C = C_w(x, t) \quad \text{at } y = 0, \\ u \rightarrow U_\infty(x, t), C \rightarrow C_\infty(x, t), T \rightarrow T_\infty(x, t), \quad \text{as } y \rightarrow \infty, \end{aligned} \quad (5)$$

where u and v are the velocity components along the x and y directions, respectively; t is time; T_w and C_w are temperature and nanoparticle volume fraction at the plate surface, respectively; U_∞ , T_∞ , and C_∞ are the free stream velocity, temperature, and nanoparticle volume fraction, respectively; ρ is the density; α is the thermal diffusivity; ν is the kinematic viscosity; c_p is the specific heat at constant pressure; σ is the electrical conductivity; $\tau = (\rho C)_p / (\rho C)_f$ is the ratio of the effective heat capacitance of the nanoparticle to that of the base fluid; D_B is the Brownian motion coefficient; D_T is the thermophoretic coefficient; $T_{0,\infty}$ is the constant free stream temperature; and C is the nanoparticle volume fraction.

Following Ibrahim and Shankar [19], we prescribe the free stream velocity and the stratified free stream and plate surface temperature and nanoparticles concentration together with the unsteady magnetic field as follows:

$$\begin{aligned} U_\infty(x, t) = \frac{ax}{1 - \lambda t}, T_w(x, t) = T_{0,\infty} + \frac{bx}{1 - \lambda t}, T_\infty(x, t) = T_{0,\infty} + \frac{cx}{1 - \lambda t}, \\ B(t) = \frac{B_0}{\sqrt{1 - \lambda t}}, C_w(x, t) = C_{0,\infty} + \frac{mx}{1 - \lambda t}, C_\infty(x, t) = C_{0,\infty} + \frac{nx}{1 - \lambda t}, \end{aligned} \quad (6)$$

where $\lambda t < 1$, a , b , c , m , and n are positive constants and λ with dimension $(\text{time})^{-1}$ is unsteadiness frequency parameter. Positive values of b , c , n , and m correspond to the assisting flows and their negative values correspond to opposing flows. In order to simplify the mathematical analysis of the problem, we introduce the following dimensionless variables:

$$\begin{aligned} \eta = \sqrt{\frac{a}{\nu(1 - \lambda t)}} y, \psi = \sqrt{\frac{a\nu}{(1 - \lambda t)}} x f(\eta), \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_{0,\infty}} = \frac{T - T_{0,\infty}}{T_w - T_{0,\infty}} - \frac{cx}{(1 - \lambda t)(T_w - T_{0,\infty})}, \\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_{0,\infty}} = \frac{C - C_{0,\infty}}{C_w - C_{0,\infty}} - \frac{nx}{(1 - \lambda t)(C_w - C_{0,\infty})}, \end{aligned} \quad (7)$$

where η is the similarity variable and ψ is the stream function defined as

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \quad (8)$$

Substituting Eqs. (7) and (8) into Eqs. (1) and (4), we obtain following system of similarity equations:

$$f''' + ff'' - f'^2 - A\left(f' + \frac{\eta}{2}f'' - 1\right) - Ha(f' - 1) + 1 = 0 \quad (9)$$

$$\begin{aligned} \theta'' - A Pr \left(\frac{\eta \theta'}{2} + \theta \right) + Pr(f\theta' + \beta_1 - f'\theta - f'\beta_1) \\ + Pr Ec[(f'')^2 + Ha(f' - 1)^2] + N_b \theta' \phi' + N_t (\theta')^2 = 0 \end{aligned} \quad (10)$$

$$\phi'' - A Le \left(\frac{\eta \phi'}{2} + \phi \right) + Le(f\phi' + \beta_2 - f'\phi - f'\beta_2) + \frac{N_t}{N_b} \theta'' = 0, \quad (11)$$

subject to the boundary conditions

$$\begin{aligned} f(0) = 0, f'(0) = 0, \theta(0) = 1 - \beta_1, \phi(0) = 1 - \beta_2, \\ f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0, \end{aligned} \quad (12)$$

where primes denote differentiation with respect to η and Ha is Hartmann number, Pr is Prandtl number, N_b is the Brownian motion parameter, N_t is the thermophoresis parameter, Ec is Eckert number, Le is Lewis number, A is the unsteadiness parameter, β_1 is the thermal stratification parameter, and β_2 is the solutal stratification parameter.

The above parameters are defined as follows:

$$\begin{aligned} Ha = \frac{\sigma B_0^2}{\alpha \rho}, Pr = \frac{\nu}{\alpha}, N_b = \frac{\tau D_B (C_w - C_{0,\infty})}{\nu}, N_t = \frac{\tau D_T (T_w - T_{0,\infty})}{T_{0,\infty} \nu}, \\ Ec = \frac{U_\infty^2}{C_p (T_w - T_{0,\infty})}, Le = \frac{\nu}{D_m}, A = \frac{\lambda}{a}, \beta_1 = \frac{c}{b}, \beta_2 = \frac{n}{m}. \end{aligned} \quad (13)$$

The significant physical quantities of engineering and industrial interest in this study are the skin friction coefficient C_f , Nusselt number Nu , and Sherwood number Sh defined, respectively, as

$$C_f = \frac{\tau_w}{\rho u_\infty^2}, Nu = \frac{x q_w}{k(T_w - T_{0,\infty})}, Sh = \frac{x q_m}{D_B(C_w - C_{0,\infty})}, \quad (14)$$

where τ_w , q_w , and q_m are the skin friction, surface heat flux, and the surface mass flux, respectively, defined by

$$\tau_w = \mu \frac{\partial u}{\partial y} \Big|_{y=0}, q_w = -k \frac{\partial T}{\partial y} \Big|_{y=0}, q_m = -D_B \frac{\partial C}{\partial y} \Big|_{y=0}. \quad (15)$$

where μ and k are the dynamic viscosity and the thermal conductivity, respectively.

Using Eqs. (15) into (14), we get

$$Re_x^{1/2} C_f = f''(0), Re_x^{-1/2} Nu = -\theta'(0), Re_x^{-1/2} Sh = -\phi'(0), \quad (16)$$

which are the local skin friction C_f , local Nusselt number Nu , and local Sherwood number Sh , respectively, and $Re_x = U_\infty x / \nu$ is the local Reynolds number.

Numerical procedure

The system of nonlinear differential Eqs. (9)–(11) with the boundary conditions (12) has been solved numerically using shooting technique coupled with the fourth-order Runge–Kutta method and a modified version of the Newton–Raphson algorithm.

Let

$$x_1 = f, x_2 = f', x_3 = f'', x_4 = \theta, x_5 = \theta', x_6 = \phi, x_7 = \phi'. \quad (17)$$

Then the set of higher order boundary value equations with their respective boundary conditions is reduced to a system of first-order differential equation with appropriate initial conditions, respectively,

$$\begin{aligned} x_1' &= x_2, \\ x_2' &= x_3, \\ x_3' &= -x_1x_3 + x_2^2 + A\left(x_2 + \frac{\eta}{2}x^3 - 1\right) + Ha(x_2 - 1) - 1, \\ x_4' &= x_5, \\ x_5' &= APr\left(\frac{\eta x_5}{2} + x_4\right) - Pr(x_1x_5 + \beta_1 - x_2x_4 - x_2\beta_1) \\ &\quad - PrEc[x_3' + Ha(x_2' - 1)] - N_bx_5x_7 - N_tx_5^2, \\ x_6' &= x_7, \\ x_7' &= ALe\left(\frac{\eta x_7}{2} + x_6\right) + Le(x_1x_7 + \beta_2 - x_2x_6 - x_2\beta_2) \\ &\quad - \frac{N_t}{N_b}\left\{ APr\left(\frac{\eta x_5}{2} + x_4\right) - Pr(x_1x_5 + \beta_1 - x_2x_4 - x_2\beta_1) \right. \\ &\quad \left. - PrEc[x_3' + Ha(x_2' - 1)] - N_bx_5x_7 - N_tx_5^2 \right\}, \end{aligned} \quad (18)$$

subject to the initial conditions

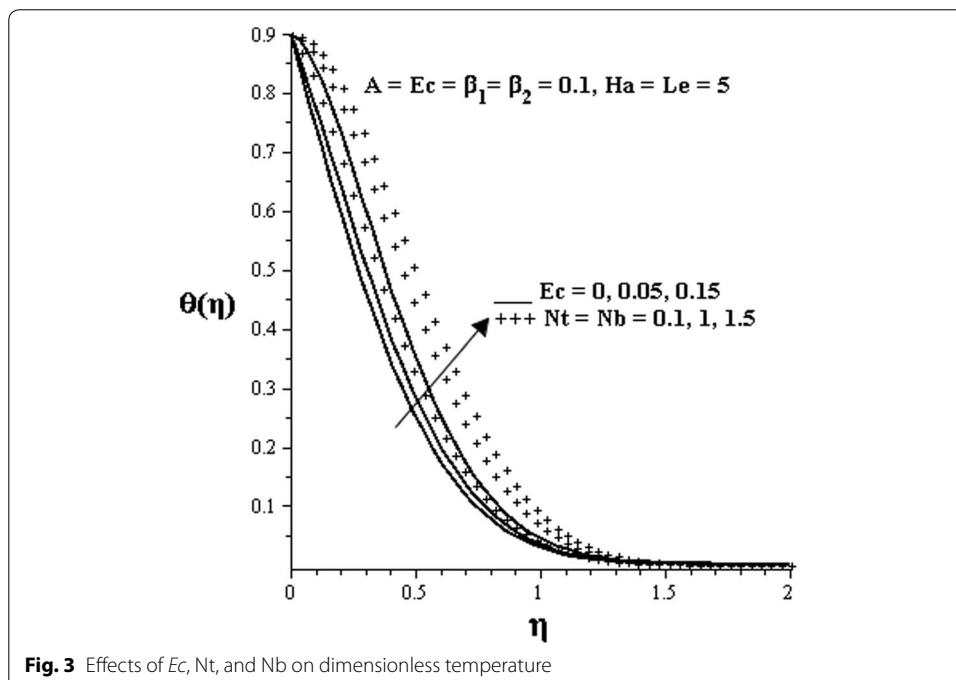
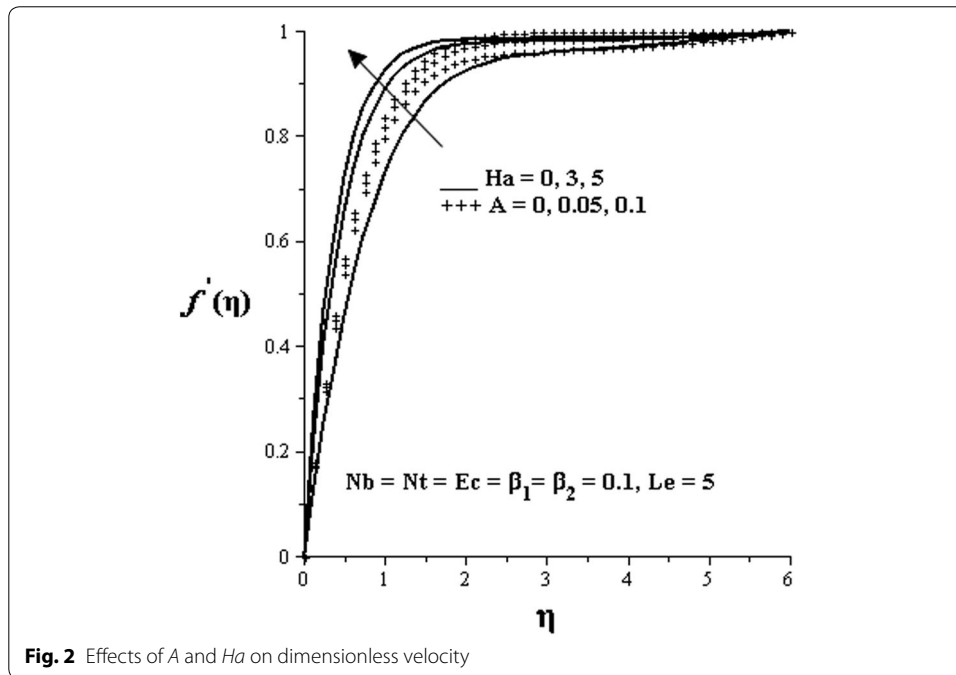
$$\begin{aligned} x_1(0) &= 0, x_2(0) = 0, x_3(0) = s_1, x_4(0) = 1 - \beta_1, \\ x_5(0) &= s_2, x_6(0) = 1 - \beta_2, x_7 = s_3, \end{aligned} \quad (19)$$

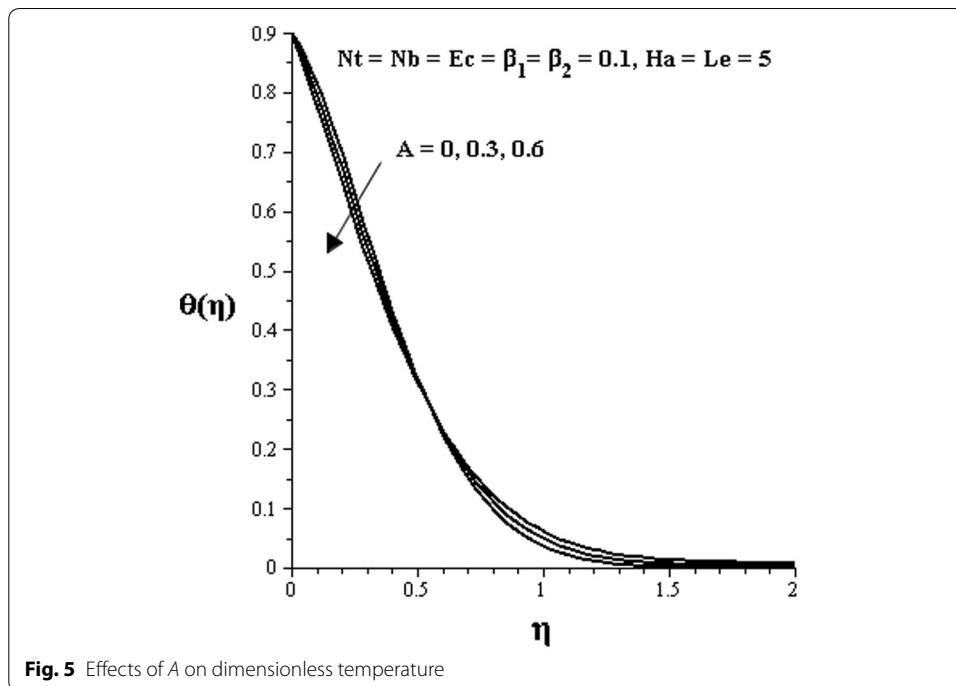
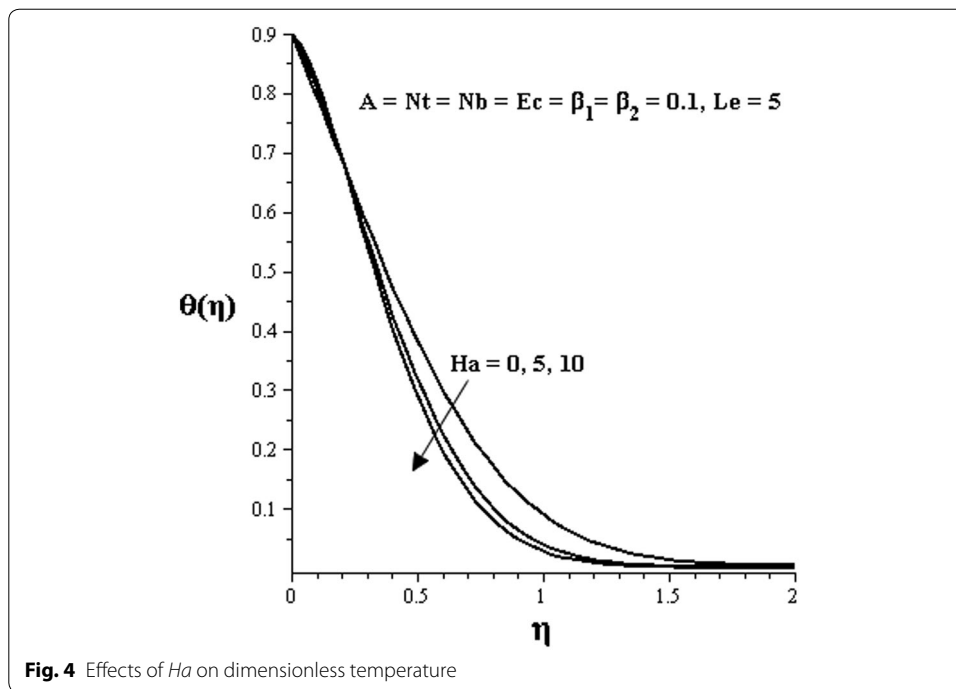
By applying the shooting method, the unspecified initial conditions S_1 , S_2 , and S_3 in (19) are assumed and (18) integrated numerically as an initial valued problem to a given terminal point. The accuracy of the assumed missing initial conditions was checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If differences exist, improved values of the missing initial conditions are obtained and the process repeated. The method is programmed in MAPLE with the step size of $\Delta\eta = 0.001$ selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The maximum value of η_∞ to each group of parameters was determined when the values of the unknown boundary conditions at $\eta = 0$ do not change to successful loop with error less than 10^{-7} .

Results and discussion

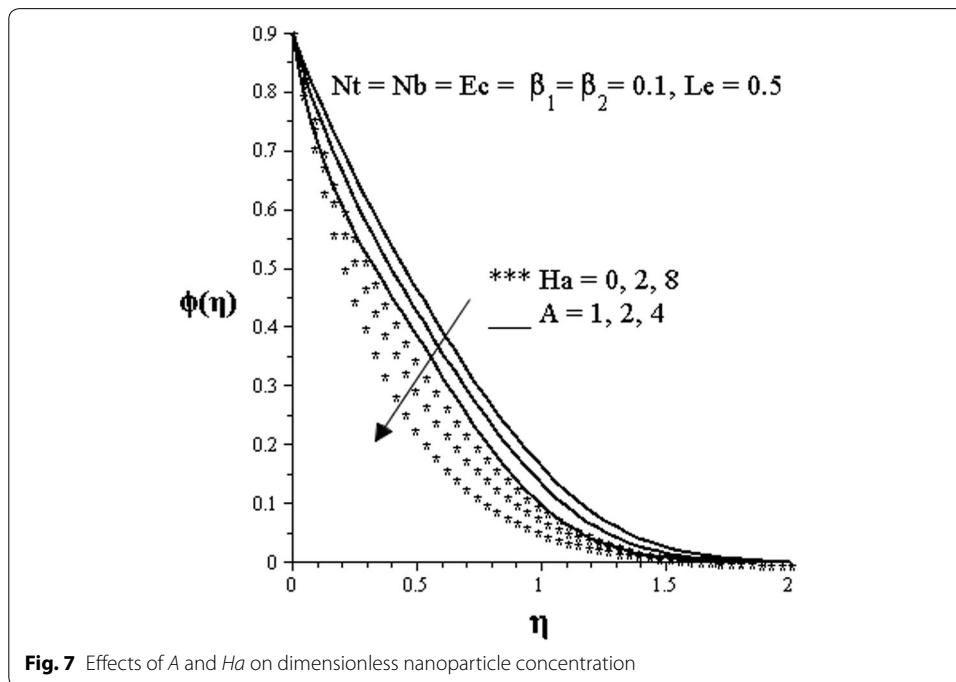
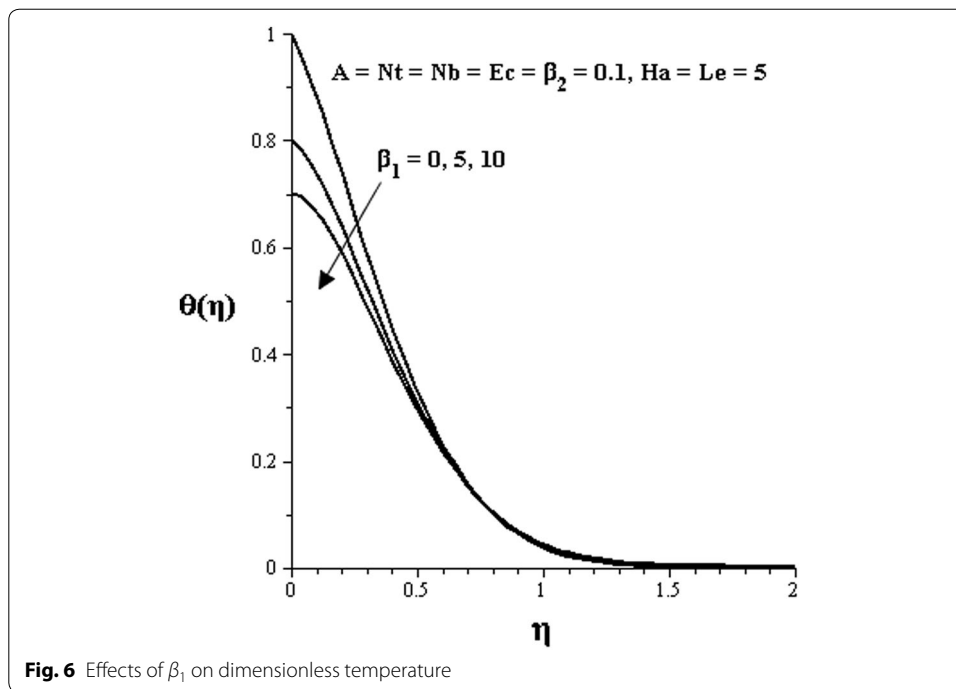
From the numerical computation process, the effects of the pertinent parameters, namely, unsteadiness parameter A , Lewis number Le , Brownian parameter N_b , thermophoresis parameter N_t , Hartmann number Ha , Eckert number Ec , thermal stratification parameter β_1 , and solutal stratification parameter β_2 on the fluid velocity, temperature, nanoparticles volume as well as the skin friction coefficient, the local

Nusselt number, and the local Sherwood number are illustrated graphically (Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14) and analysed quantitatively highlighting their industrial and engineering applications. In order to ascertain the validity of our numerical procedure, the special case of heat transfer in flow of a convective fluid in the absence of buoyancy force, flow unsteadiness, magnetic field, and double stratification effect (i.e.



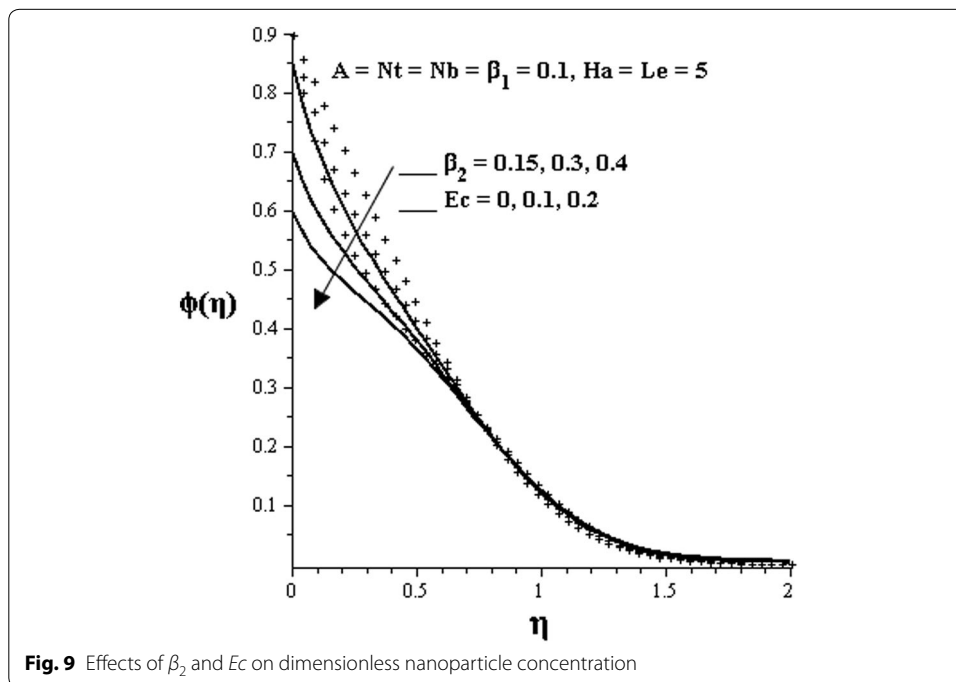
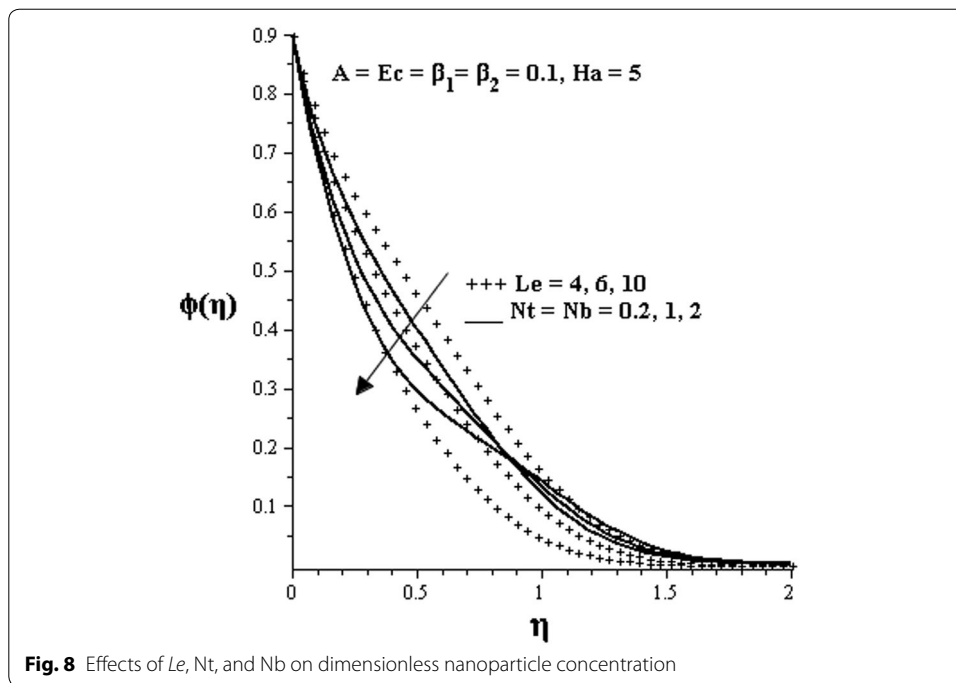


$A = \beta_1 = \beta_2 = Ha = \phi = 0$) over a surface can be compared with that of Aman et al. [20], as shown in Table 1 and excellent agreement was established. This favourable comparison lends confidence to the numerical results reported subsequently.



Velocity profiles

Figure 2 shows the effect of unsteadiness and magnetic field strength on the nanofluids' velocity profiles. It can be seen that there is a gradual increase in the fluid velocity, from zero at the plate surface to a maximum value as it approaches the free stream far away from the plate surface thus satisfying the boundary conditions. It is observed



that both the fluid velocity and the hydrodynamic boundary layer thickness decrease as the unsteadiness parameter (A) and Hartmann number (Ha) increases. It is well known that application of a transverse magnetic field orthogonal to the flow direction has a tendency to create a drag force known as Lorentz force which tends to resist the fluid flow and thus reducing its velocity.

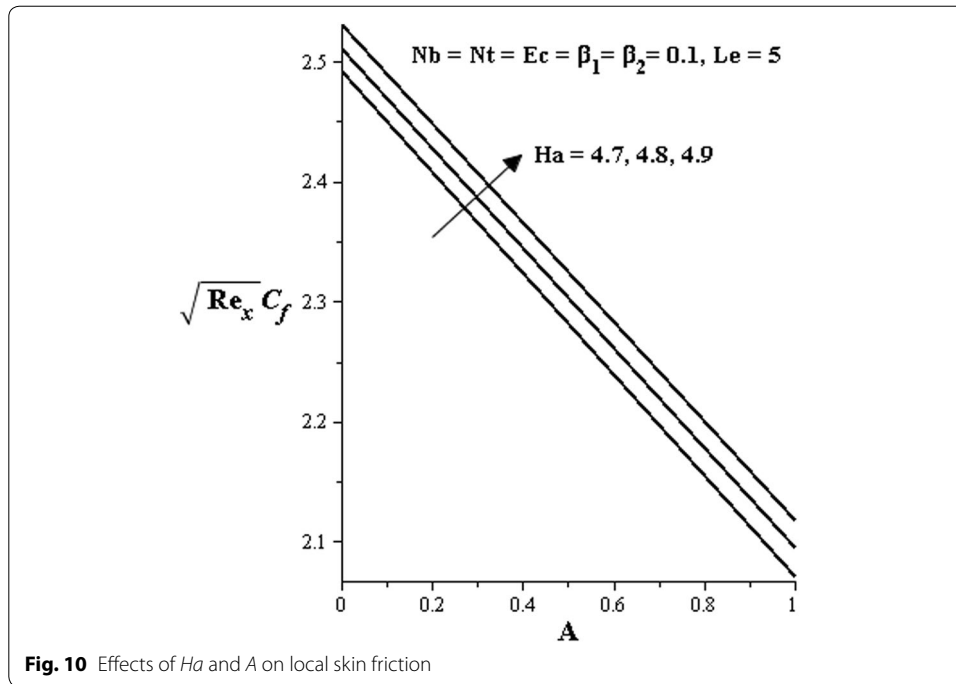


Fig. 10 Effects of Ha and A on local skin friction

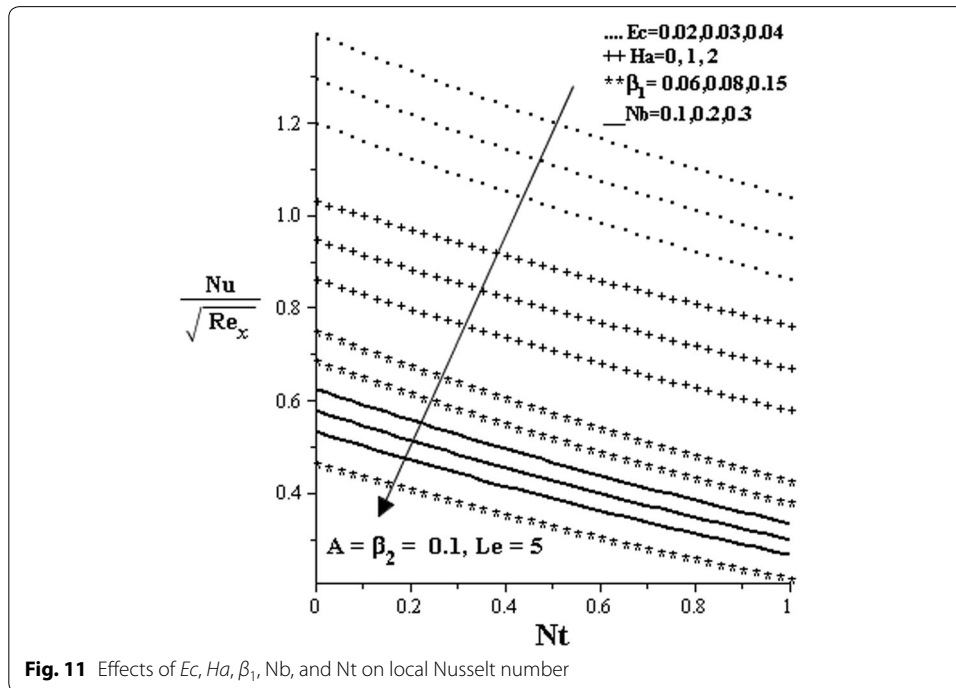


Fig. 11 Effects of Ec , Ha , β_1 , Nb , and Nt on local Nusselt number

Temperature profiles

Figures 3, 4, 5, 6 show the effects of various parameters on the temperature profile. Generally, the temperature is the maximum at the plate surface but decreases exponentially to zero far away from the plate surface satisfying the free stream conditions. As shown in

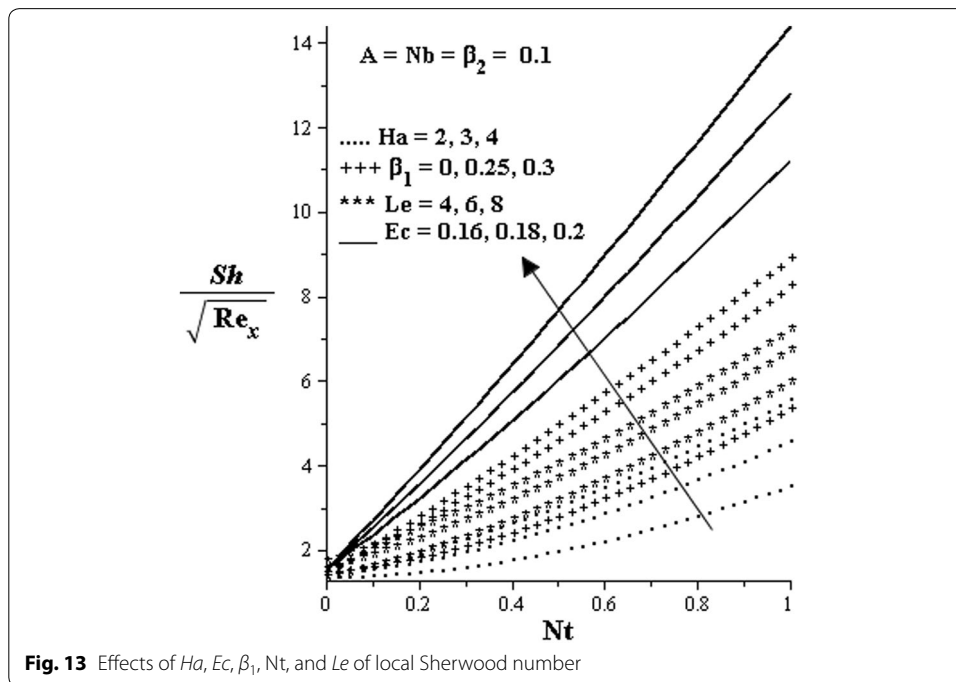
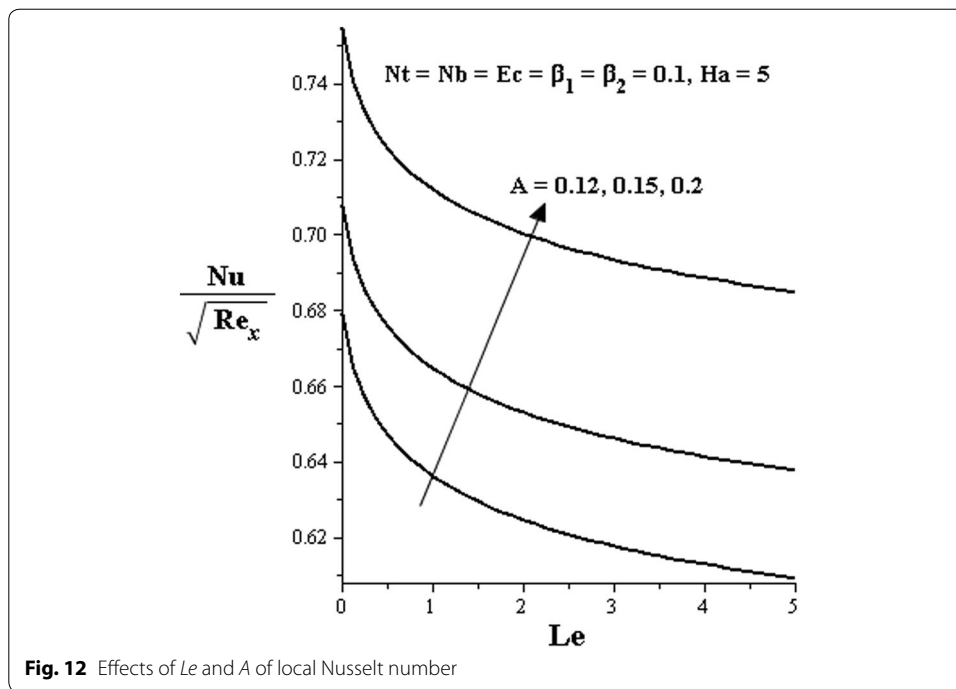


Fig. 3, an increase in Brownian parameter N_b , thermophoresis parameter N_t , and Eckert number Ec leads to an increase in both the fluid temperature and the thermal boundary layer thickness. This can be attributed to the additional heating due to viscous dissipation, friction, and rapid collision of the nanoparticles as a result of Brownian motion and thermophoresis. Figure 4 shows that increasing Hartmann number Ha initially increases

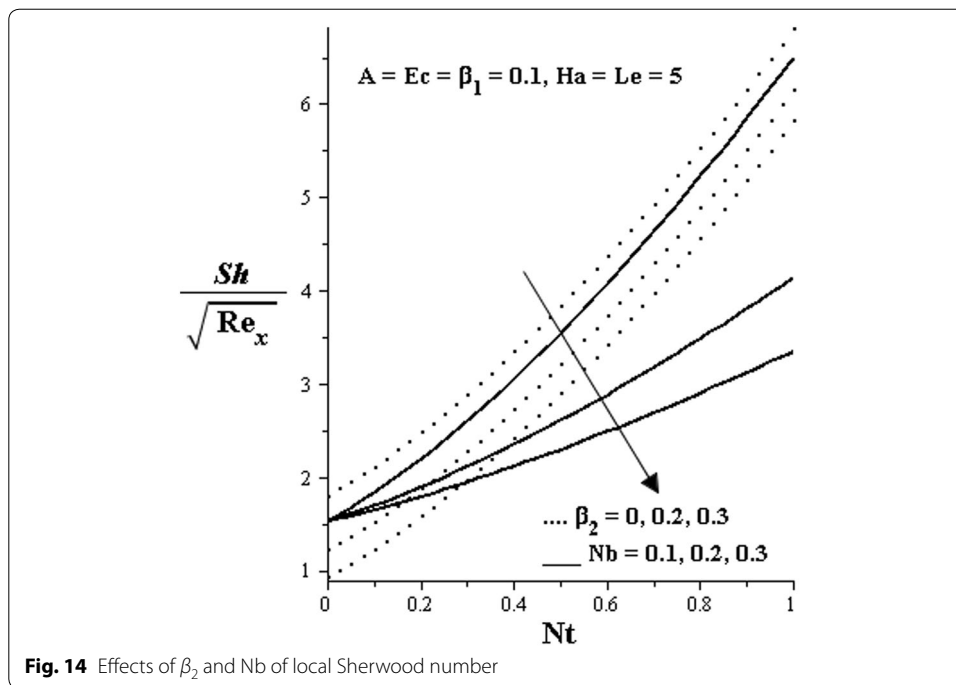


Table 1 Computations showing comparison with Aman et al. [20] for different values of Pr when $A = \beta_1 = \beta_2 = Ha = \varphi = 0$

Pr	$f''(0)$ Amani et al. [20]	$-\theta'(0)$ Amani et al. [20]	$f''(0)$ present	$-\theta'(0)$ present
0.1	2.50997	0.26815	2.50997	0.26815
0.3	2.50997	0.40338	2.50997	0.40338
0.7	2.50997	0.52965	2.50997	0.52965
6.2	2.50997	0.59014	2.50997	0.59014

the fluid temperature at the plate surface, but as the fluid moves from the plate surface the temperature decreases. The increase in temperature is due to ohmic heating as a result of the magnetic effect. On the other hand, as illustrated in Figures 5 and 6, there is a decrease in both the temperature and the thermal boundary layer thickness with an increase in thermal stratification parameter β_1 and unsteadiness parameter A . When the thermal stratification effect is taken into consideration, the effective temperature difference between the plate and the ambient fluid decreases; therefore, the temperature is reduced.

Nanoparticle concentration profiles

Figures 7, 8, 9 show the effects of various parameters on the nanoparticle concentration profile. The nanoparticle concentration is the highest at the plate surface but decreases exponentially to zero free stream value satisfying the boundary conditions. Unlike in the case of temperature, a decrease in the nanoparticle concentration is noted with an increase in unsteadiness parameter A , Lewis number Le , Brownian parameter N_b ,

thermophoresis parameter N_t , Hartmann number Ha , Eckert number Ec , and solutal stratification parameter β_2 .

Effects of parameter variations on C_f , Nu , Sh

Figures 10, 11, 12, 13, 14 depict the various pertinent parameters at the plate surface for the skin friction coefficient, the local Nusselt number (rate of heat transfer), and the local Sherwood number with variation in the values of thermophysical parameters embedded in the problem. From Fig. 10, it can be seen that increasing both the unsteadiness parameter A and Hartmann number Ha leads to an increase in the skin friction coefficient. This is as expected since both parameters causes an overshoot of the fluid towards the plate surface. From Fig. 11, it is noted increasing the Lewis number Le , Brownian parameter N_b , thermophoresis parameter N_t , Hartmann number Ha , Eckert number Ec , and thermal stratification parameter β_1 leads to a decrease in the local Nusselt number which implies that there is reduced heat transfer rate at the plate surface. As seen earlier, the same parameters led to an increase to the fluid temperature; thus, it is expected that with increased temperature within the fluid the rate of heat transfer from the plate to the fluid will be reduced. On the other hand, an increase in the unsteadiness parameter A increases the local Nusselt number, as observed in Fig. 12. It is noted in Figs. 13 and 14 that increasing the Hartmann number Ha , Eckert number Ec , Lewis number Le , thermophoresis parameter N_t , and thermal stratification parameter β_1 increases the Sherwood number, while a decrease in the local Sherwood number is realized by increasing the Brownian parameter N_b and solutal stratification parameter β_2 . It well known that at high Lewis number the nanoparticle concentration is low, since the concentration at the plate surface is higher than in the fluid, mass transfer from the plate to the fluid then occurs.

Conclusions

The combined effects of thermal stratification, solutal stratification, Brownian motion, and thermophoresis on an unsteady hydromagnetic boundary layer flow of a nanofluid over a flat surface are numerically investigated. Using a similarity variable, the governing nonlinear partial differential equations were obtained and transformed to a set of nonlinear differential equations. Numerical results are obtained, then presented graphically for the velocity, temperature, and nanoparticle concentration profiles as well as skin friction coefficient, reduced Nusselt number, and reduced Sherwood number. Our results reveal, among other conclusions, the following:

- The fluid overshoots towards the plate surface with increasing A and Ha , thus reducing both the fluid velocity and the hydrodynamic boundary layer thickness.
- An increase in Ec , Nt , Nb , and Ha yields an increase in both the thermal boundary layer thicknesses and temperature, whereas the reverse is noted with increases in A and β_1 .
- A decrease in the nanoparticle concentration is noted with increase in A , Le , Nb , Nt , Ha , Ec , and β_2 .
- Increasing A and Ha increase the skin friction.

- A decrease in Nusselt number is realized with increased Ec , Ha , Nb , and β_1 , whereas it increases with increasing values of A .
- The local Sherwood number increases with increase in Ha , β_1 , Le , Nt , and Ec but reduces with increased β_2 and Nb .

Abbreviations

(u, v) : velocity components in x - and y -direction; A : unsteadiness parameter (λ/a); B_0 : magnetic field strength (T); C : nanoparticle volume fraction; C_f : skin friction coefficient ($\tau_w/U_\infty^2\rho$); $C_{0,\infty}$: constant free stream volume fraction; D_B : Brownian diffusion coefficient; D_T : thermophoresis diffusion coefficient; Ec : Eckert number ($U_\infty^2/C_p(T_w - T_{0,\infty})$); Ha : magnetic parameter ($\sigma B_0^2 B_0^2/\alpha\rho$); k : thermal conductivity (W/mK); Le : Lewis number (v/D_m); N_b : Brownian motion parameter ($\tau D_B(C_w - C_{0,\infty})/v$); N_t : thermophoresis parameter ($\tau D_T(T_w - T_{0,\infty})/T_{0,\infty}v$); Pr : Prandtl number (v/α); Nu : Nusselt number ($xq_w/k(T_w - T_{0,\infty})$); Sh : Sherwood number ($xq_m/D_B(C_w - C_{0,\infty})$); T : temperature of fluid (K)

Greek symbols

ψ : stream function; η : similarity variable; β_1 : thermal stratification parameter; β_2 : solutal stratification parameter; ν : kinematic viscosity of fluid (kg/m s); ρ : density of fluid (kg/m³); ϕ : dimensionless concentration function; α : thermal diffusivity (W/m k); σ : electrical conductivity of fluid (S/m); θ : dimensionless temperature; $(c_p)_f$: heat capacity of fluid (J/kg K); $(c_p)_p$: heat capacity of nanofluid; τ : parameter defined by $(\rho C)_p/(\rho C)_f$; μ : dynamic viscosity of fluid (kg/m s)

Subscripts

∞ : condition at the free stream; w : condition at the surface

Authors' contributions

Both authors were actively involved in the formulation of the problem, literature review, and simulations. Both authors read and approved the final manuscript.

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Competing interests

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Ethics approval and consent to participate

This study does not involve human participants, human data, or human tissue.

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